# Chiral bands in nuclei

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A system is chiral if it is distinguishable from its mirror image.

Spontaneous formation of handedness or chirality in
molecular physics,
the characterization of elementary particles,
optical physics, etc.

Presence of noncoplanar 3 angular-momentum vectors (in the body-fixed system)

$\longrightarrow$ chirality and intrinsic triaxial shape in nuclei.

S.Frauendorf and J.Meng, Nucl. Phys. A617, 131 (1997)

Expected observation :

Two almost degenerate $\Delta I = 1$ rotational bands with the same parity, which appear only after a sufficient amount of collective rotation.

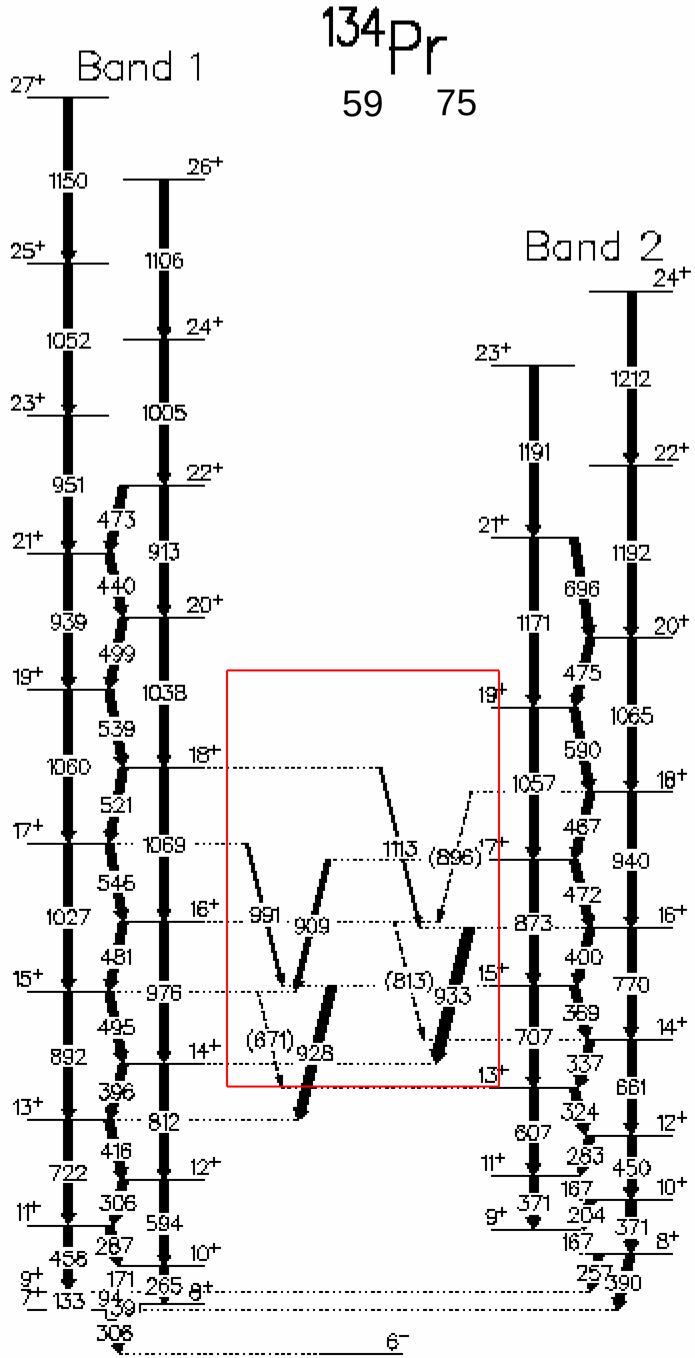
ex. observed in some odd-odd nuclei in $A \sim 130$ region ?

Experimental data

C.M.Petrache et al.,
N.P. A597,106 (1996),

and

GS2K009 Collaboration,
K.Starosta et al., AIP Conf. Proc. No. 610, (AIP, New York, 2002), p.815.




may be regarded as
[$^{134}_{58}Ce_{76}$ x one proton-particle
x one neutron-hole]

Bands are defined so that they are connected by Strong I → I-2 E2 transitions.

# *Systematics of partner bands in odd–odd A~130 nuclei.*

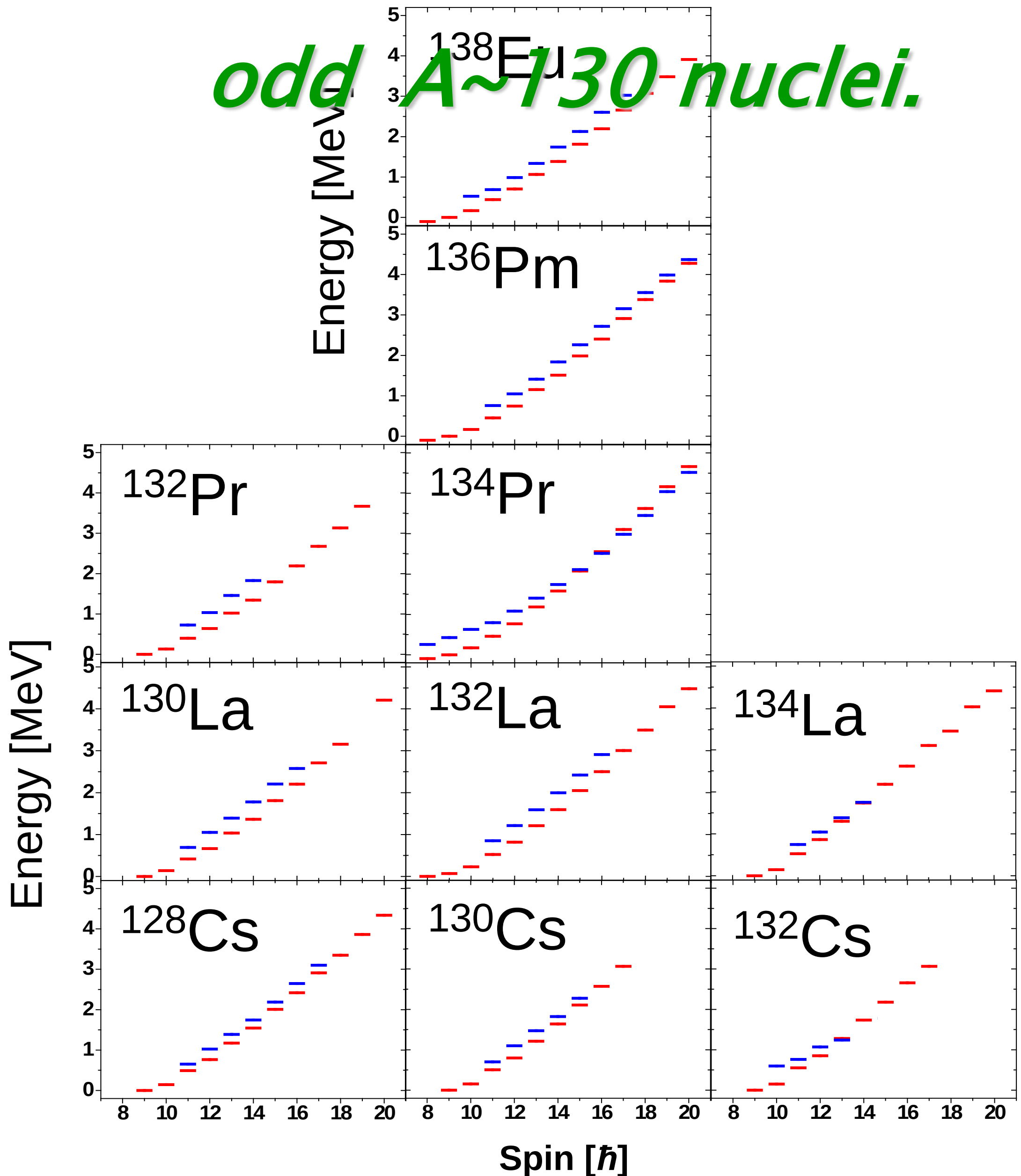
Z=63

Z=61

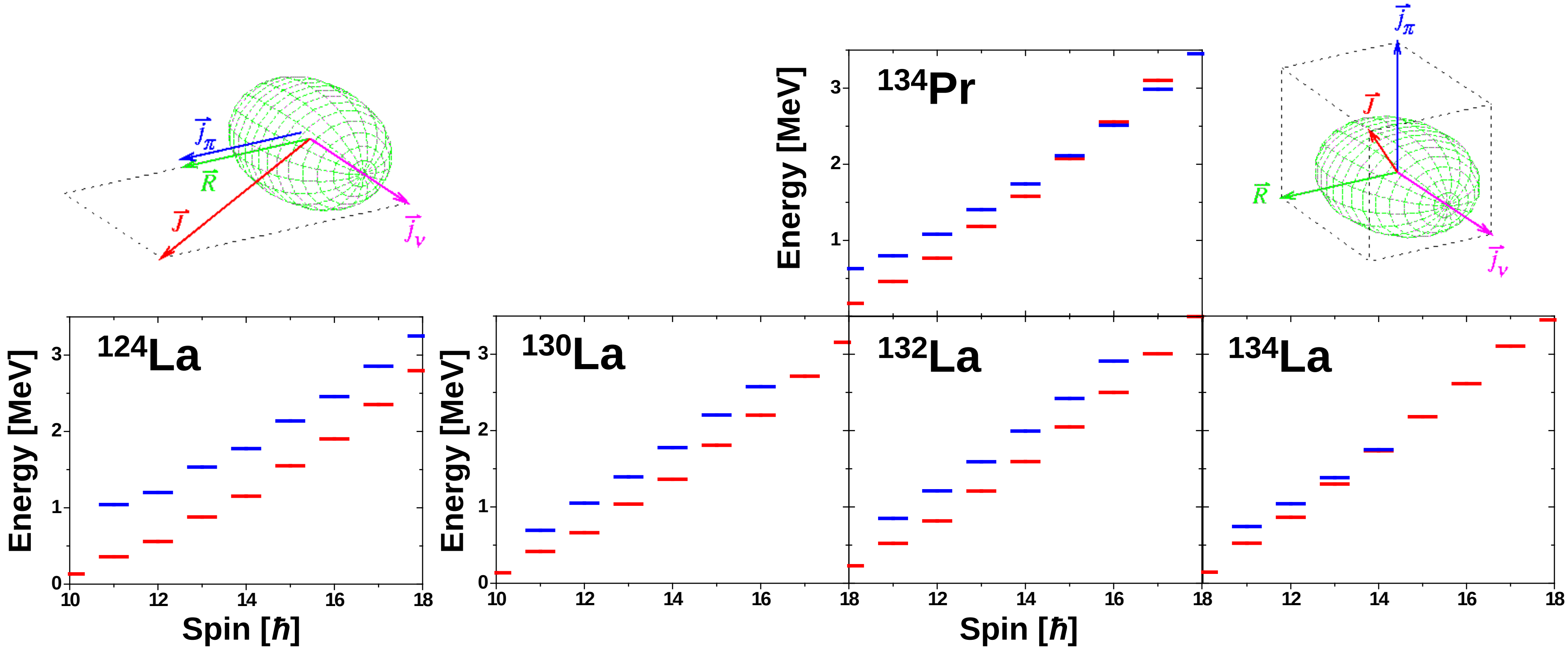
Z=59

Z=57

Z=55



# *Band separation for Z=57 as a function of neutron number .*



## Ellipsoidal shape for small deformation in the body-fixed (intrinsic) system

Radius

$$R(\theta,\varphi) = R_0 \{1 + \beta \left[ Y_{20} \cos\gamma + \frac{1}{\sqrt{2}} ( Y_{22} + Y_{2-2} ) \sin\gamma \right] \}$$

$$= R_0 \{ 1 + \beta \sqrt{\frac{5}{16\pi}} [(3 \cos^2\theta - 1) \cos\gamma + \sqrt{3} \sin\gamma \sin^2\theta \cos 2\varphi ] \}$$

To describe all possible quadrupole-deformed shapes, deformation parameters $\beta \geq 0$ and $0 \leq \gamma \leq 60°$ are sufficient. Ex. $\gamma$=0°, 120°, 240° give prolate spheroids with 3, 1 and 2 axes as symmetry axes.

## Triaxially-quadrupole deformed part of one-body potential

$$V(r,\theta,\phi) = k(r)\beta \left\{ Y_{20} \cos\gamma + \frac{1}{\sqrt{2}} (Y_{22} + Y_{2-2}) \sin\gamma \right\} \qquad (1)$$

For a particle in single-j shell (high-j shell such as $h_{11/2}$ , $i_{13/2,}$ …) the potential (1) is equivalent to

$$(1) \longrightarrow \frac{\kappa}{j(j+1)} \left\{ \left[ 3 j_3^2 - j(j+1) \right] \cos\gamma + \sqrt{3} ( j_1^2 - j_2^2 ) \sin\gamma \right\} \qquad (2)$$

because

$$\frac{\langle j,\Omega | Y_{20} | j,\Omega \rangle}{\langle j,\Omega+2 | Y_{22} | j,\Omega \rangle} \propto \frac{\langle j,\Omega | 3 j_3^2 - j(j+1) | j,\Omega \rangle}{\langle j,\Omega+2 | j_+^2 | j,\Omega \rangle} \qquad \text{etc.}$$

where Ω denotes the body-fixed (i.e. intrinsic) 3-component of one-particle angular-momentum j .

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For deformation $\gamma$ = 90º ⟵ preferred by moderately rotating even-even nuclei

(2) $\propto$ $( j_1^{\,2} - j_2^{\,2} )$ for one-particle in the single-j-shell. Thus, [$\vec{j}$ ∥ (2-axis) ] for energy-min.

(2) $\propto$ $( j_2^{\,2} - j_1^{\,2} )$ for one-hole in the single-j-shell. Thus, [ $\vec{j}$ ∥ (1-axis)] for energy-min.

Deviation from spherical shape

$$\delta R \propto \sum_{\mu} Y_{\lambda\mu}{}^{*}(\theta,\varphi)\, \alpha_{\lambda\mu}$$

where μ : component in lab. system

Quadrupole deformation

$$\alpha_{2\mu} = \sum_{\nu} a_{2\nu}\; D_{\mu\nu}{}^{2}(\Omega)$$

$a_{2\nu}$ : deformation parameters in body-fixed system

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Ellipsoid :

$$[\, a_{21} = a_{2-1} = 0 \quad and \quad a_{22} = a_{2-2} \,]$$

five $a_{2\nu} \leftrightarrow$ three Euler angles to fix intrinsic system relative to lab. system

while two $a_{22}$ $and$ $a_{20}$ determine the shape of ellipsoid.

$$a_{20} = \beta \cos \gamma$$

$$a_{22} = 2^{-1/2}\, \beta\, \sin\gamma$$

β and ϒ are the deformation parameters to determine the shape of ellipsoid.

## For triaxial shape $\gamma = 90°$ --- maximum triaxiality

1) Three radii of ellipsoid have the relation

$$R_1\ (= R_0 + \delta R\ ) \ > \ R_3\ (= R_0\ ) \ > \ R_2\ (= R_0 - \delta R)$$

where

$$\delta R = R_0 \sqrt{\frac{5}{4\pi}}\ \beta\ \frac{\sqrt{3}}{2}$$

Ex. $\delta R_1 = R\left(\frac{\pi}{2}, 0\right) - R_0 = R_0 \sqrt{\frac{5}{4\pi}}\ \beta\ \frac{\sqrt{3}}{2}$

Ex. Quadrupole moment along 3-axis is

$$Q_0 \propto (2{R_3}^2 - {R_1}^2 - {R_2}^2\ ) \sim O(\ (\delta R)^2\ )$$

2) Moments of inertia

2a) Rigid moments of inertia

$$\Im_2 > \Im_3 > \Im_1 \qquad \because)\ (\Im_{rig})_1 \propto ({R_2}^2 + {R_3}^2\ )$$

2b) Hydrodynamical irrotational moments of inertia

$\vec{\nabla} \times \vec{v} = rot\ (\vec{v}\ ) = 0 \ \longrightarrow$ velocity field $\vec{v}$ is irrotational

$$\Im_3 > \Im_2 = \Im_1\ (= \frac{1}{4}\Im_3\ ) \qquad \because)\ (\Im_{irrot})_k \propto {R_0}^2\, \beta^2\, sin^2(\gamma - \frac{2\pi}{3}k)$$

Ex. $(\Im_{irrot})_1 \propto \dfrac{({R_2}^2 - {R_3}^2)^2}{({R_2}^2 + {R_3}^2\ )}$

In the rotational band based on the ground state of even-even nuclei
[$\gamma$-dependence of $\Im_{calc} \approx$ that of $\Im_{irrot}$ ] in the presence of pair correlation.

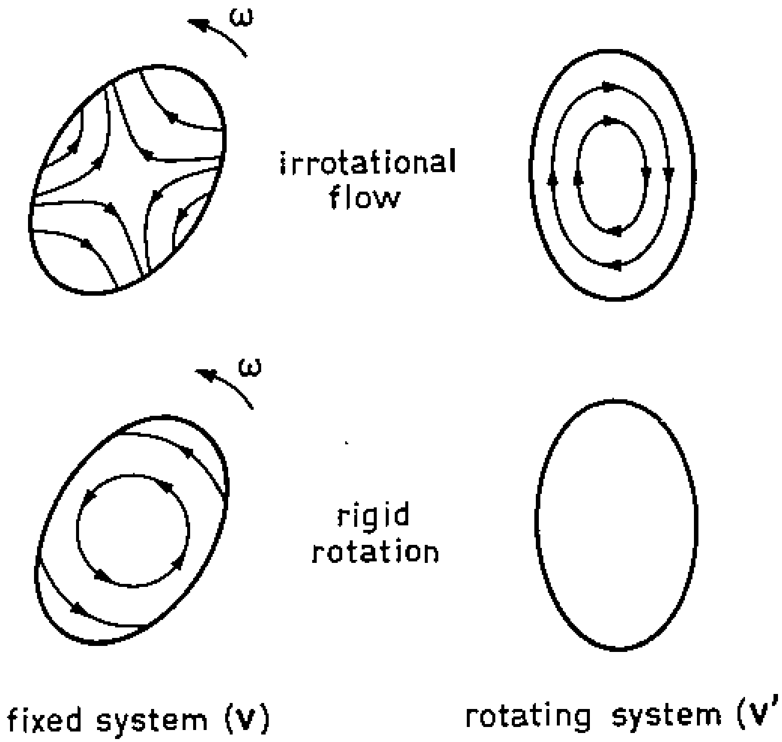



**Figure 6A-2** Flow patterns for rotation of ellipsoidal body.

For constructing a given angular-momentum, even-even core-nuclei with $\mathfrak{I}_{irrot}$ prefer to rotate about the intermediate (note $R_1 > R_3 > R_2$) 3-axis.

In deformed nuclei with Z ≈ 59 and N ≈ 75
the neutron Fermi-level $\lambda_n$ ≈ lowest-lying π = neg level coming from $1h_{11/2}$ shell, while
the proton Fermi-level $\lambda_p$ ≈ highest-lying π = neg level coming from $1h_{11/2}$ shell.

One particle levels coming from high-j shell (h11/2-shell in this case) remain almost pure high-j structure under both quadrupole deformation and rotation.

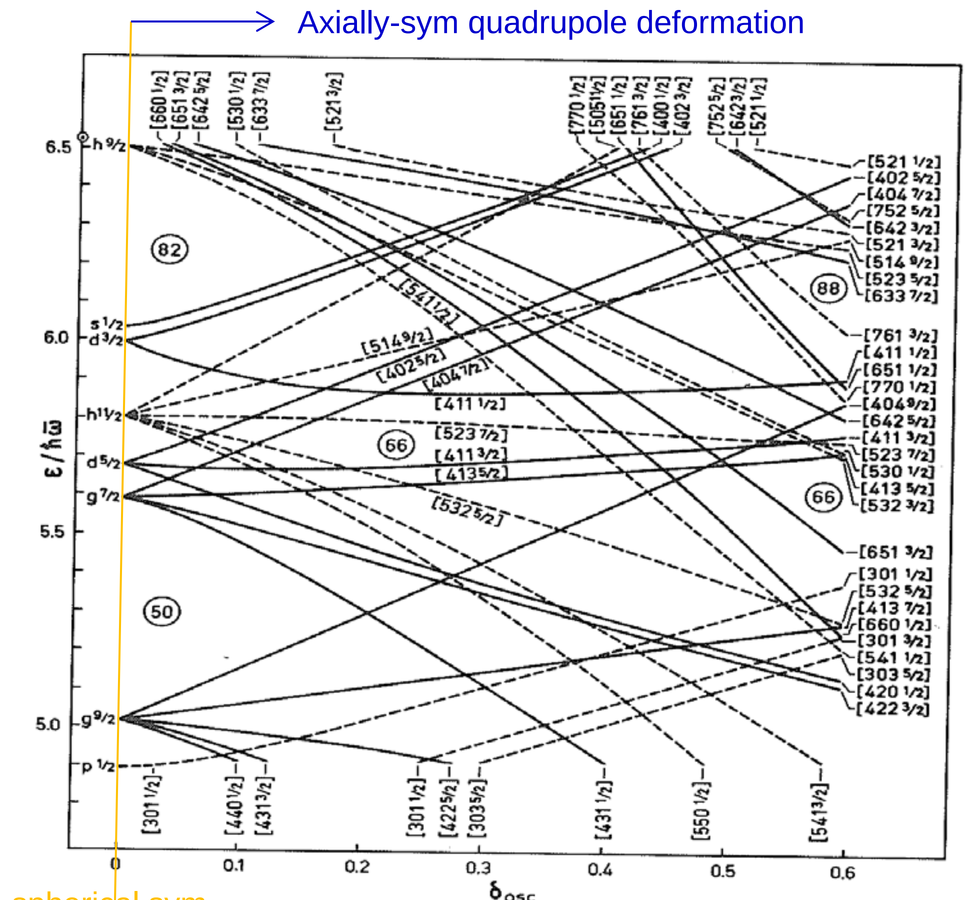
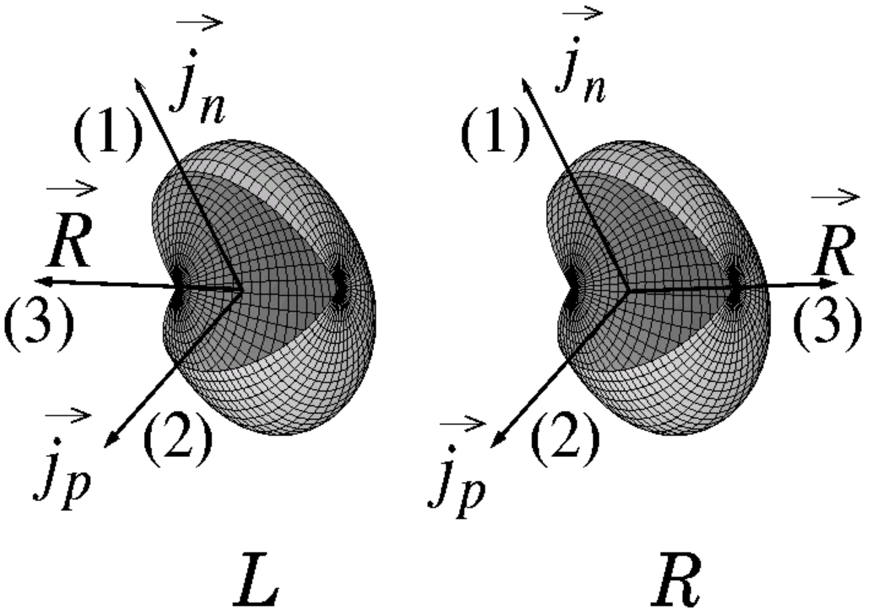


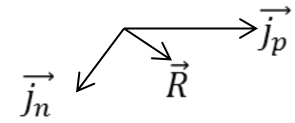

**Figure 5-2** Proton orbits in prolate potential ($50 < Z < 82$). The spectra in this and the following figures (Figs. 5-2 to 5-5) are taken from C. Gustafson, I. L. Lamm, B. Nilsson, and S. G. Nilsson, *Arkiv Fysik* **36**, 613 (1967). The orbits are labeled by the asymptotic quantum numbers $[Nn_3\Lambda\Omega]$. Levels with even and odd parity are drawn with solid and dashed lines, respectively. (Erratum: The orbit [301 3/2] is incorrectly labeled [301 1/2] at bottom of figure.)

That means, some set of low-lying levels of $^{134}$Pr with Z=59 and N=75 may be expressed by the configuration of one proton in $h_{11/2}$ shell and one neutron hole in $h_{11/2}$ shell coupled to the even-even core $^{134}$Ce with Z=58 and N=76. → Particle-rotor model

To construct cheaply a given total angular-momentum $\vec{I} = \vec{R} + \vec{j_n} + \vec{j_p}$

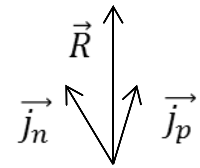



a) $I$ : small (R: small)



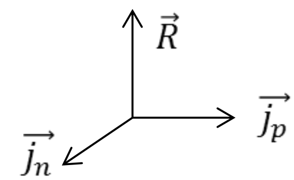

$\vec{R}$ : ang-mtm of collective rotation of even-even core and lies on the plane defined by $\vec{j_p}$ and $\vec{j_n}$

b) $I$ : large (R: large)




$\vec{j}_p$ and $\vec{j}_n$ *align to* $\vec{R}$

c) $I$ : medium (R: medium)




Chiral geometry ?

1-axis : longest axis of the triaxial shape : neutron-hole in a high-$j_n$ shell

2-axis : shortest axis : proton-particle in a high-$j_p$ shell

3-axis : intermediate axis : core angular-momentum (hydrodynamical moments of inertia)

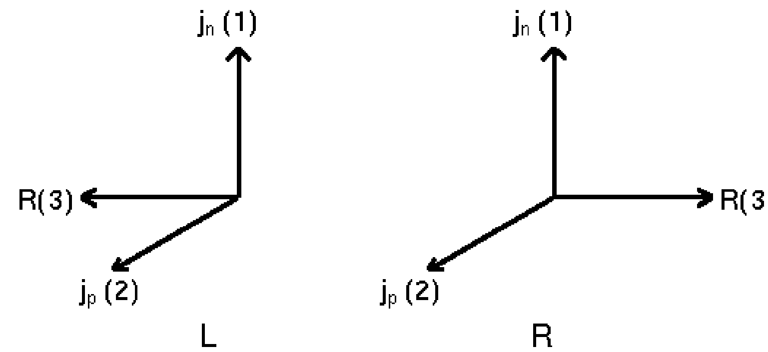
Total Hamiltonian is invariant under L ↔ R

$$\vec{I} = \vec{R} + \vec{j}_n + \vec{j}_p$$

When chiral geometry is realized, observed two chiral degenerate states are,

$$|I+\rangle = \frac{1}{\sqrt{2}}\left(|IL\rangle + |IR\rangle\right)$$

$$|I-\rangle = \frac{i}{\sqrt{2}}\left(|IL\rangle - |IR\rangle\right)$$




If no tunneling between L and R, then $|I+\rangle$ and $|I-\rangle$ are degenerate.

For *I* » 1 ( |***R***| » 1)

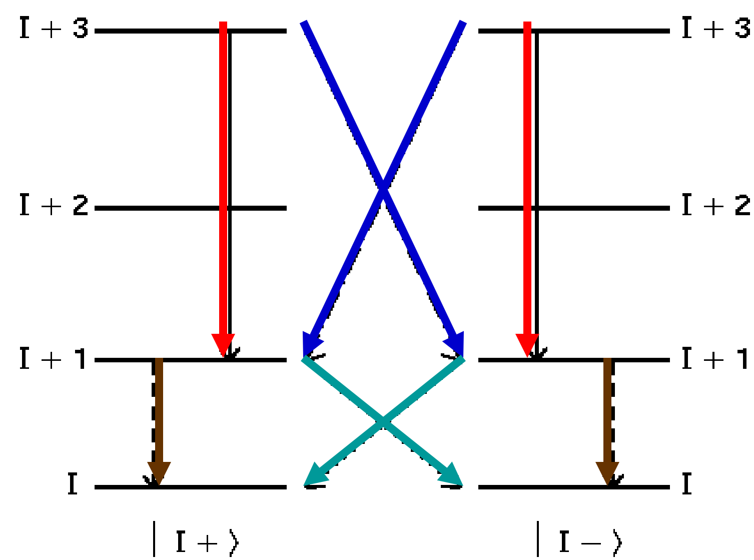
$$\langle IL|E2|IR\rangle \approx 0$$

$$\langle IL|M1|IR\rangle \approx 0$$

Then, for *EM* = *E2* or *M1*

$$B(EM; I'+ \to I+) \approx B(EM; I'- \to I-)$$

$$B(EM; I'+ \to I-) \approx B(EM; I'- \to I+)$$



Experimental data

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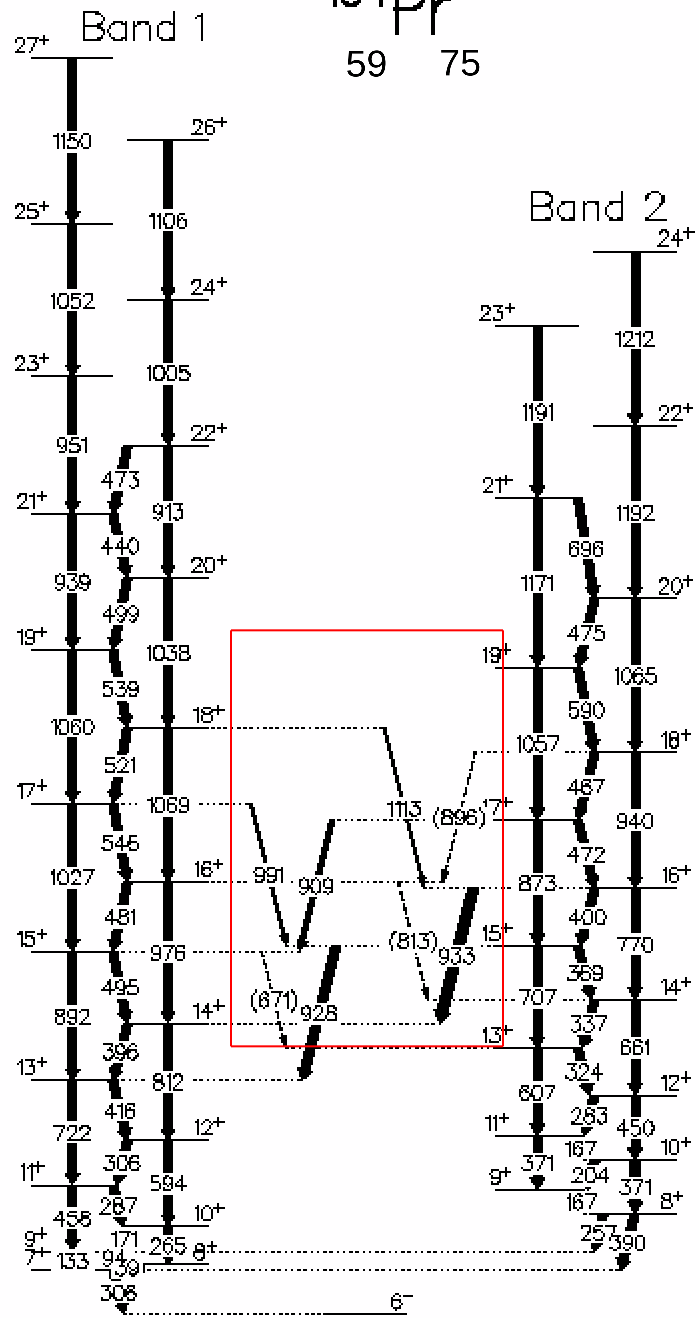
$^{134}_{59}Pr_{75}$



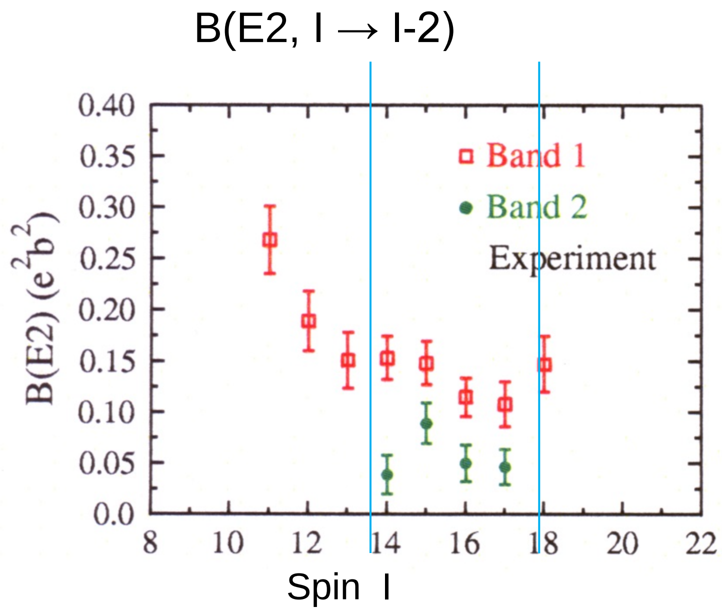

may be regarded as
[$^{134}_{58}Ce_{76}$ x one proton-particle
x one neutron-hole]

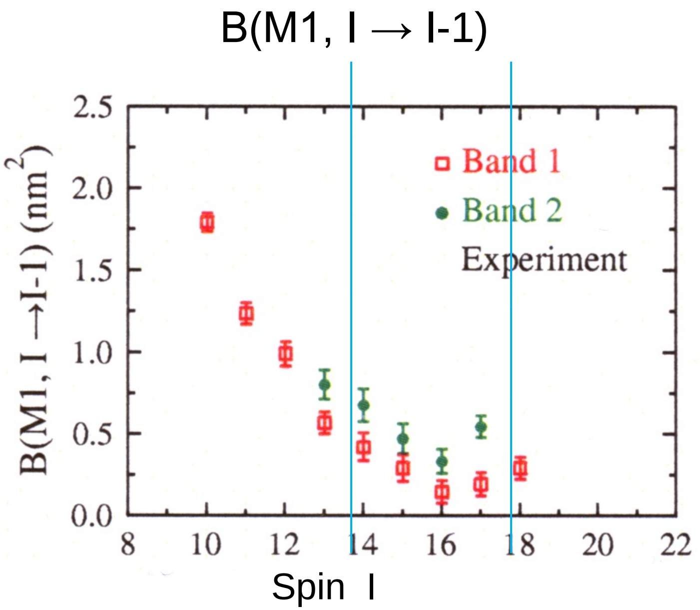
Chiral geometry ⟶
two bands are degenerate

How about the similarity of *B(E2)* and *B(M1)* values in two rotational bands ?

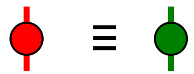
$^{134}_{59}Pr_{75}$ Data from: D.Tonev et al., PRL **96**, 052501 (2006).

Exp data

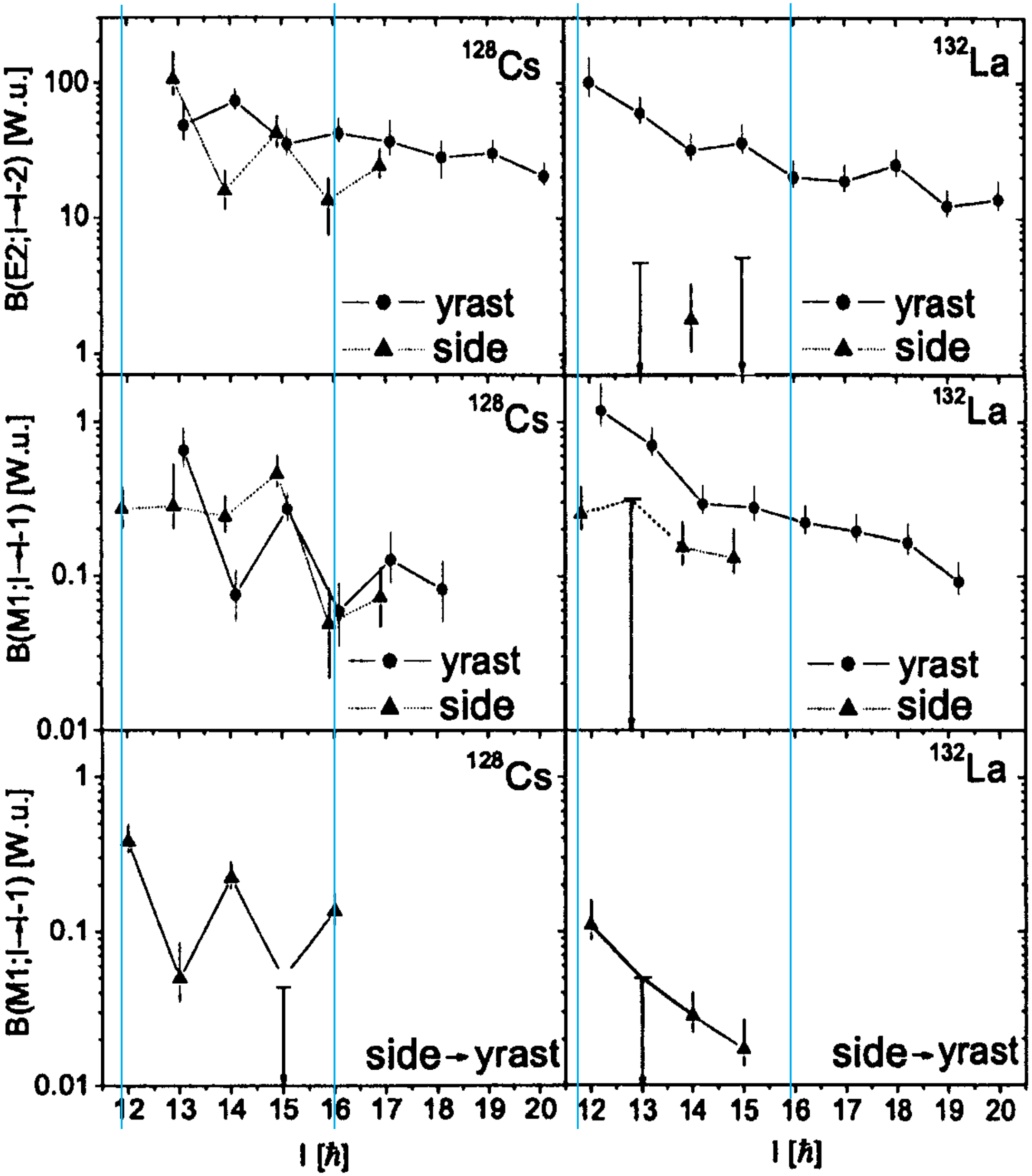



In the chiral twin bands,



OBS. Around $I = 14 \sim 18$ the two $\Delta I = 1$ rotational bands are nearly degenerate .




E.Grodner et al., PRL **97**,172501 (2006)

$\pi\, h_{11/2}\ (\nu\, h_{11/2})^{-1}$ configuration

in $^{128}_{55}Cs_{73}$ and $^{132}_{57}La_{75}$

Measured B(E2) and B(M1) values within respective chiral pair-bands

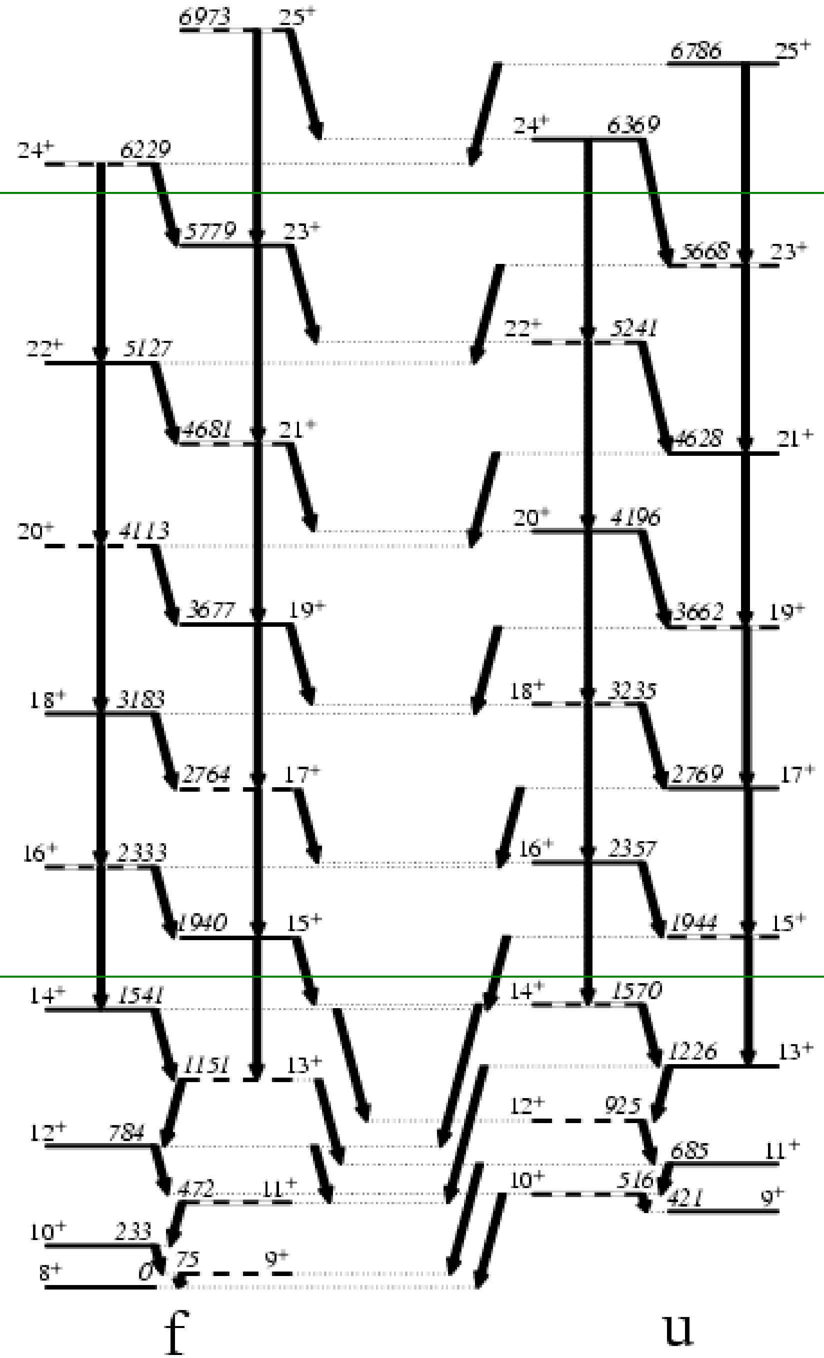
In the region of $12 \leq \boldsymbol{I} \leq 16$ *two* $\Delta\boldsymbol{I}$ =1 rotational bands are parallel ($\Delta E$ ~ *a few hundred keV*).

# Particle-rotor calc in a specific model

A=130, Z=55

The $\pi h_{11/2}$ and $\nu h_{11/2}^{-1}$ configuration coupled to a γ=–30° (or +90° ) triaxial core.

Chiral region ?




In the region of $14 \leq I \leq 23$,

1) t wo bands are nearly degenerate;
2) more specified selection rules for electro-magnetic transitions pin down chiral geometry.

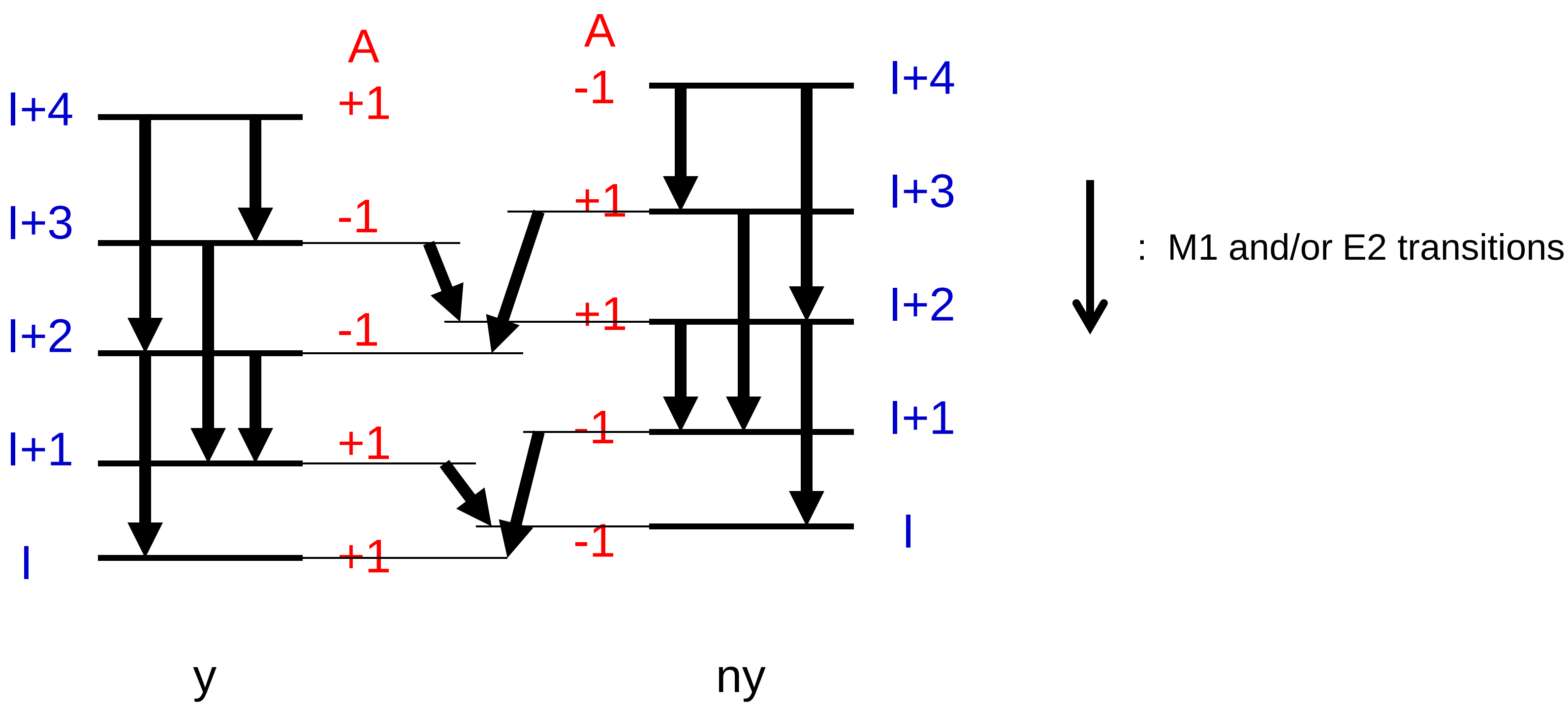
In a specific model of odd-odd nuclei;

T.Koike, K.Starosta and I.H., PRL 93, 172502 (2004)

- triaxial core with $\gamma$ = +90°  ,  assuming $J_{hydro}(\gamma)$
- one proton-particle in j-shell
- one neutron-hole in j-shell    *namely,* $j_p = j_n$

one finds a quantum number (A = ± 1)

→ selection-rule for electromagnetic transitions in chiral pair bands
in the case of chiral geometry



Odd-odd nuclei (such as $^{134}_{59}Pr_{75}$)

Particle-rotor Hamiltonian $H = H_{rot} + T_\pi + V_{sp}{}^{\pi} + T_\nu + V_{sp}{}^{\nu}$

Total angular momentum $\vec{I} = \vec{R} + \vec{j}_p + \vec{j}_n$

$I$, $j_p$ and $j_n$ are good quantum-numbers.

Even-even core nucleus with γ=+90° , assuming $J_{hydro}$

$$H_{rot} = \sum_k \frac{\hbar^2}{2J_k} R_k^2 \Rightarrow \frac{\hbar^2}{8J_0}\left(R_3^2 + 4(R_1^2 + R_2^2)\right)$$

One proton-particle in $j$ (=$h_{11/2}$) shell

$$V_{sp}^{\pi} \propto (j_{p1}^2 - j_{p2}^2)$$

One neutron-hole in $j$ (=$h_{11/2}$) shell

$$V_{sp}^{\nu} \propto (j_{n2}^2 - j_{n1}^2)$$

The interaction between the rotor and particles is expressed by the potential $V_{sp}$ anisotropic in the body-fixed system.

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For intermediate ***I*** one may approximately expect (chiral geometry)

$\vec{j}_p$ || ± 2-axis, (s-axis) $\vec{j}_n$ || ± 1-axis, (l-axis) $\vec{R}$ || ± 3-axis (i-axis)

# Invariance properties of $H=H_{rot}+V_{sp}^{\pi}+V_{sp}^{\nu}$ → quantum-number $A$

Irrespective of chirarlty

1) Ellipsoid has $D_2$ *symmetry* $\rightarrow R_3 = 0, \pm 2, \pm 4, \ldots\ldots$

2) Invariant under the operation $A$ consisting of

2.1) rotation $\exp\left(i(\frac{\pi}{2})R_3\right)$ or $\exp\left(i(\frac{3\pi}{2})R_3\right)$

combined with

2.2) charge symmetry [$C : n \leftrightarrow p$]

$D_2$ symmetry: invariant under
$1,\ R_1(\pi),\ R_2(\pi),\ R_3(\pi)$

$$e^{-i\pi R_3} D_{MR_3}{}^{R}(\Omega) = (-1)^{R_3}\ D_{MR_3}{}^{R}\ (\Omega)$$

$$H_{rot} \propto (R_3{}^2 + 4(R_1{}^2 + R_2{}^2))$$

$$V_{sp}{}^{\pi} + V_{sp}{}^{\nu} \propto (j_{p1}{}^2 - j_{p2}{}^2) + (j_{n2}{}^2 - j_{n1}{}^2)$$

Defining $C$= +1 symmetric for $n \leftrightarrow p$
-1 anti-symmetric for $n \leftrightarrow p$ ,

eigenstates of ***H*** have quantum-number $A = \pm 1$ .

$A$=+1 state : components with $R_3 = 0, \pm 4, \pm 8, \ldots$ and $C$ = +1 or
components with $R_3 = \pm 2, \pm 6, \ldots$ and $C$ = -1

$A$=-1 state : components with $R_3 = 0, \pm 4, \pm 8, \ldots$ and $C$=-1 or
components with $R_3 = \pm 2, \pm 6, \ldots$ and $C$=+1

## Selection rule for E2 and M1 transitions

1) To obtain non-zero values of

B(E2; core contribution only) $\rightarrow$ $\Delta C=0$ $\quad\because$) n and p are spectators.

$\Delta R_3 = \pm 2$ since $\gamma=+90^\circ$ $\quad\because$) $Q_0 = 0$

then,

B(E2; $i \rightarrow f$) = 0 for $A_i = A_f$

| | |
|---|---|
| A=+1 state: | C=+1 & $R_3$ = 0, ±4, ±8, … |
| | C=-1 & $R_3$ = ±2, ±6, … |
| A=-1 state:: | C=-1 & $R_3$ = 0, ±4, ±8, … |
| | C=+1 & $R_3$ = ±2, ±6, … |

2) $$(M1)_\mu = \sqrt{\frac{3}{4\pi}}\frac{e\hbar}{2mc}\left((g_\ell - g_R)\,\ell_\mu + (g_s^{eff} - g_R)\,s_\mu\right)$$

where $g_\ell - g_R = 0.5$ (-0.5)

$g_s^{eff} - g_R = 2.848$ (-2.792)

} M1 operator is nearly isovector.

for p (n) and $g_R = 0.5$ and $g_s^{eff} = 0.6\ g_s^{free}$ are used.

Then, B(M1; $i \rightarrow f$) ≈ 0 for $C_i = C_f$ . $\quad\because$) Contributions by n and p almost cancel

Since *M1* cannot change $|\Delta R_3| \geq 2$ ,

[ B(M1; $i \rightarrow f$) with $A_i = A_f$ ] « [ B(M1; $i \rightarrow f$) with $A_i \neq A_f$ ]

# In chiral geometry

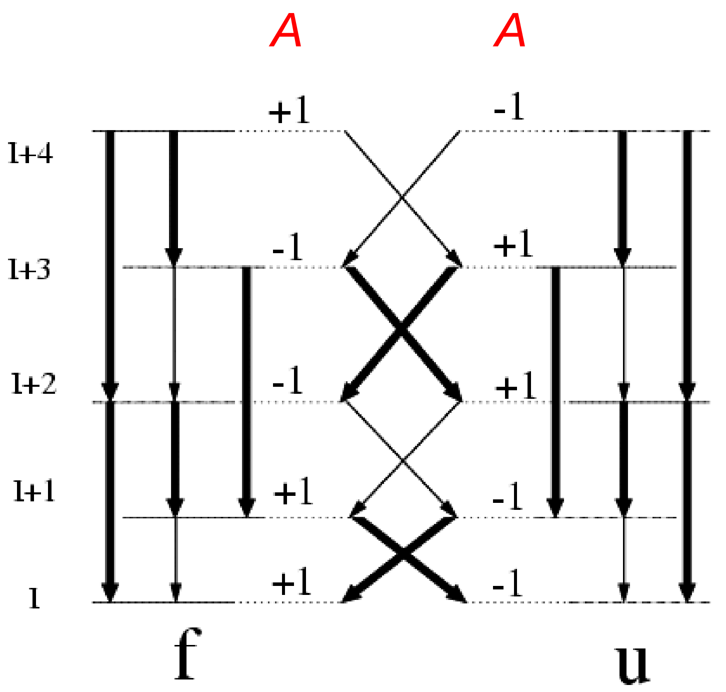
$C\,|L> \;\propto\; |R>$
$C\,|R> \;\propto\; |L>$

while keeping the direction of $\vec{R}$ unchanged.

namely,

$A\,|L> \;\propto\; |R>$

$A\,|R> \;\propto\; |L>$

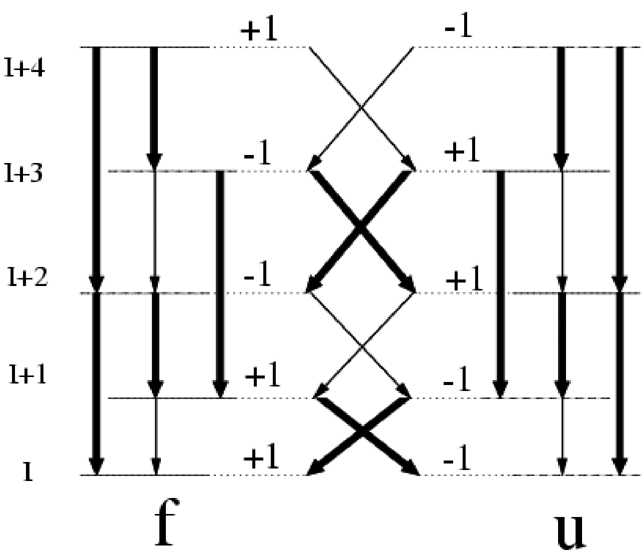


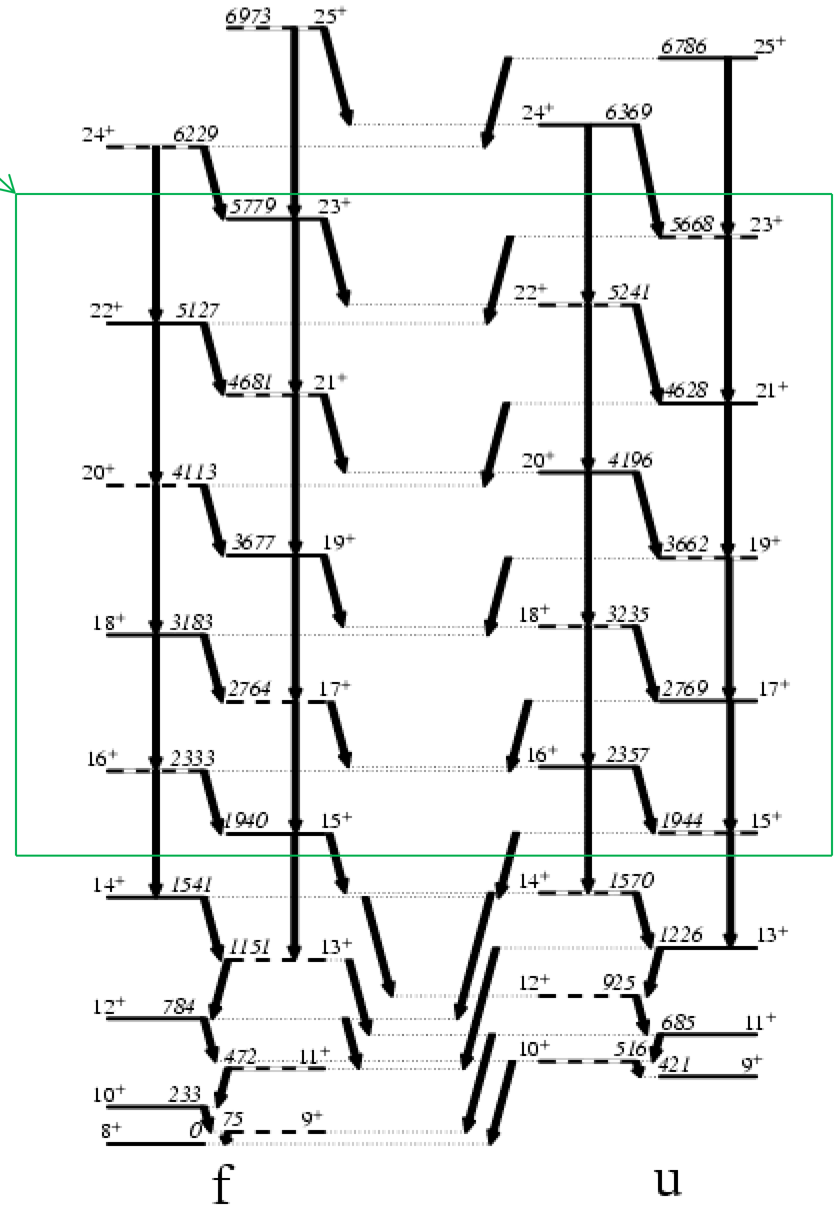

Thus, two degeneate states, $|I+>$ and $|I->$, have different eigenvalues of $A$.

$A\,|I+> = \pm\,|I+> \quad \leftrightarrow \quad A\,|I-> = \mp\,|I->$

$$|I+\rangle = \frac{1}{\sqrt{2}}\left(|IL\rangle + |IR\rangle\right)$$
$$|I-\rangle = \frac{i}{\sqrt{2}}\left(|IL\rangle - |IR\rangle\right)$$

The formation of chiral geometry in some range of total angular-momentum is numerically confirmed by the selection rule for electro-magnetic transitions in an unambiguous manner.




Calculated result of diagonalizing the particle-rotor Hamiltonian,

$H = H_{rot} + V_{sp}^{\pi} + V_{sp}^{\nu}$ with $\gamma=+90°$ .